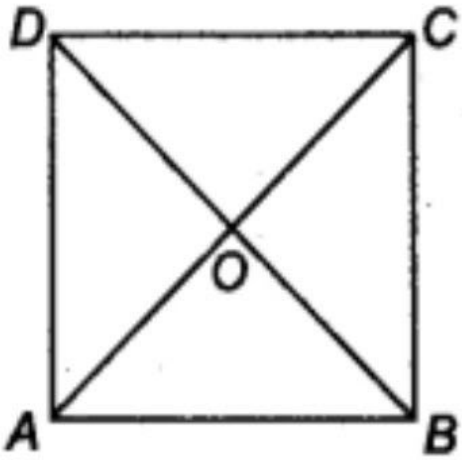
**Ex-8.1 solved exercise**
**Quadrilaterals By- Ashish jha**

**Q.4.Show that the diagonals of a square are equal and bisect each other at right angles.**

**Solution**:

Let ABCD be a square such that its diagonals AC and BD intersect at O.




(i) To prove that the diagonals are equal, we need to prove AC = BD.
In ΔABC and ΔBAD, we have
AB = BA [Common]
BC = AD [Sides of a square ABCD]
∠ABC = ∠BAD [Each angle is 90°]
∴ ΔABC ≅ ΔBAD [By SAS congruency]
AC = BD [By C.P.C.T.] …(1)

(ii) AD || BC and AC is a transversal. [∵ A square is a parallelogram]
∴ ∠1 = ∠3
[Alternate interior angles are equal]
Similarly, ∠2 = ∠4
Now, in ΔOAD and ΔOCB, we have
AD = CB [Sides of a square ABCD]
∠1 = ∠3 [Proved]
∠2 = ∠4 [Proved]
∴ ΔOAD ≅ ΔOCB [By ASA congruency]
⇒ OA = OC and OD = OB [By C.P.C.T.]
i.e., the diagonals AC and BD bisect each other at O. …….(2)

(iii) In ΔOBA and ΔODA, we have
OB = OD [Proved]
BA = DA [Sides of a square ABCD]
OA = OA [Common]
∴ ΔOBA ≅ ΔODA [By SSS congruency]

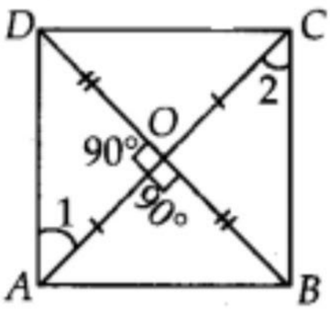
⇒ ∠AOB = ∠AOD [By C.P.C.T.] …(3)
∵ ∠AOB and ∠AOD form a linear pair.
∴∠AOB + ∠AOD = 180°
∴∠AOB = ∠AOD = 90° [By(3)]
⇒ AC ⊥ BD …(4)
From (1), (2) and (4), we get AC and BD are equal and bisect each other at right angles.

**Q.5.Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.**

**Solution:**

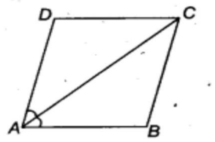
Let ABCD be a quadrilateral such that diagonals AC and BD are equal and bisect each other at right angle.

Now, in ΔAOD and ΔAOB, We have
∠AOD = ∠AOB [Each 90°]



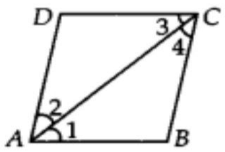

AO = AO [Common]
OD = OB [ ∵ O is the midpoint of BD]
∴ ΔAOD ≅ ΔAOB [By SAS congruency]
⇒ AD = AB [By C.P.C.T.] …(1)
Similarly, we have
AB = BC … (2)
BC = CD …(3)
CD = DA …(4)
From (1), (2), (3) and (4), we have
AB = BC = CD = DA
∴ Quadrilateral ABCD have all sides equal.
In ΔAOD and ΔCOB, we have
AO = CO [Given]
OD = OB [Given]
∠AOD = ∠COB [Vertically opposite angles]
So, ΔAOD ≅ ΔCOB [By SAS congruency]
∴∠1 = ∠2 [By C.P.C.T.]
But, they form a pair of alternate interior angles.
∴ AD || BC
Similarly, AB || DC
∴ ABCD is a parallelogram.
∴ Parallelogram having all its sides equal is a rhombus.
∴ ABCD is a rhombus.
Now, in ΔABC and ΔBAD, we have

AC = BD [Given]
BC = AD [Proved]
AB = BA [Common]
∴ ΔABC ≅ ΔBAD [By SSS congruency]
∴ ∠ABC = ∠BAD [By C.P.C.T.] ……(5)
Since, AD || BC and AB is a transversal.
∴∠ABC + ∠BAD = 180° .. .(6) [ Co – interior angles]
⇒ ∠ABC = ∠BAD = 90° [By(5) & (6)]
So, rhombus ABCD is having one angle equal to 90°.
Thus, ABCD is a square.
**Q.6.Diagonal AC of a parallelogram ABCD bisects ∠A (see figure). Show that**




**(i) it bisects ∠C also,**
**(ii) ABCD is a rhombus.**

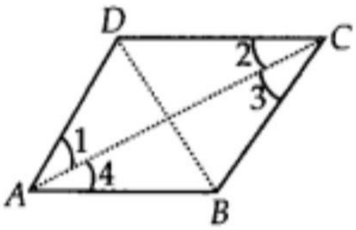
Solution:
We have a parallelogram ABCD in which diagonal AC bisects ∠A
⇒ ∠DAC = ∠BAC




(i) Since, ABCD is a parallelogram.
∴ AB || DC and AC is a transversal.
∴ ∠1 = ∠3 …(1)
[ ∵ Alternate interior angles are equal]
Also, BC || AD and AC is a transversal.
∴ ∠2 = ∠4 …(2)
[ v Alternate interior angles are equal]
Also, ∠1 = ∠2 …(3)
[ ∵ AC bisects ∠A]
From (1), (2) and (3), we have
∠3 = ∠4
⇒ AC bisects ∠C.

(ii) In ΔABC, we have
∠1 = ∠4 [From (2) and (3)]
⇒ BC = AB …(4)
[ ∵ Sides opposite to equal angles of a Δ are equal]
Similarly, AD = DC ……..(5)
But, ABCD is a parallelogram. [Given]
∴ AB = DC …(6)
From (4), (5) and (6), we have

AB = BC = CD = DA
Thus, ABCD is a rhombus.
**Q.7.ABCD is a rhombus. Show that diagonal AC bisects ∠Aas well as ∠C and diagonal BD bisects ∠B as well AS ∠D.**
**Solution:**
Since, ABCD is a rhombus.




⇒ AB = BC = CD = DA
Also, AB || CD and AD || BC
Now, CD = AD ⇒ ∠1 = ∠2 …….(1)
[ ∵ Angles opposite to equal sides of a triangle are equal]
Also, AD || BC and AC is the transversal.
[ ∵ Every rhombus is a parallelogram]
⇒ ∠1 = ∠3 …(2)
[ ∵ Alternate interior angles are equal]
From (1) and (2), we have
∠2 = ∠3 …(3)
Since, AB || DC and AC is transversal.
∴ ∠2 = ∠4 …(4)
[ ∵ Alternate interior angles are equal] From (1) and (4),
we have ∠1 = ∠4
∴ AC bisects ∠C as well as ∠A.
Similarly, we can prove that BD bisects ∠B as well as ∠D.
**Please wait for the next part... Thanks**