MOTION

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If we look around us, we find that there are number of objects which are in motion. **An object is said to be in motion if it change its position with the passage of time.** In other words, the movement of an object is known as the motion of the object.

Now observe the following bodies or objects and we will be able to understand the meaning of the term "motion". Cars, cycles, motorcycles, scooters, buses, rickshaws, trucks etc. running on the road, Birds flying in the sky, Fish swimming in water. All these objects are in motion. Very small objects like atoms and molecules and very large objects like planets, stars and galaxies are in motion.

Thus, all objects ranging from a smallest atom to the largest galaxy are in continuous motion.

TYPES OF MOTION :

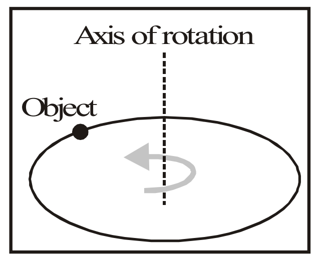
(A) Linear motion : A body has linear motion if it moves in a straight line or path.

Ex. (i) Motion of a moving car on a straight road.

(ii) Motion of a ball dropped from the roof of a building.

(B) Circular (or rotational) Motion : A body has circular motion if it moves around a fixed point.

A vertical passing through the fixed point around which the body moves is known as axis of rotation.



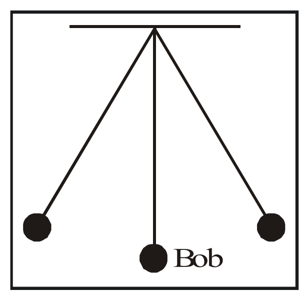
Ex. (i) Motion of an electric fan.

(ii) Motion of merry-go-round

(iii) Motion of a spinning top.

(C) Vibratory motion :

A body has vibratory motion if it moves to and fro about a fixed point.



Ex. (i) Motion of a pendulum of a wall clock.

(ii) Motion of a simple pendulum.

MOTION IN LIVING AND NON LIVING OBJECTS :

It is a common observation that all living objects, whether plants or animals can move in some way or the other. The motion in animals is more apparent than the motion in plants.

The motion in animals is called LOCOMOTION.

Plants also move but their motion is not apparent as they cannot move from place to place. Their motion takes place in parts. As a plant grows so does its roots and its leaves.

MECHANICS :

The branch of physics which deals with the motion of non-living objects in everyday life is called mechanics or Classical mechanics. It is of two types.

(i) Statics and (ii) Dynamics

STATICS : Statics deals with bodies at rest under the effect of different forces.

DYNAMICS : Dynamics deals with the bodies in motion. It is further of two types :

(i) **Kinematics** : Kinematics, which is derives from a Greek word **kinema** meaning motion, is a branch of Physics, which deals with the motion of a body without taking into account the cause of motion.

(ii) **Dynamics proper** : Dynamics proper, which is derived from a Greek word **dyna** meaning power it is a branch of Physics, which deals with the motion of bodies by taking into account the cause of motion (force).

CONCEPT OF A POINT OBJECT, REST AND MOTION

Point object : An extended object can be treated as a point object when the distance travelled by the object is much greater than its own size.

"A point object is one, which has no linear dimensions but possesses mass."

Ex. (i) Study of motion of a train travelling from Kota to New Delhi.

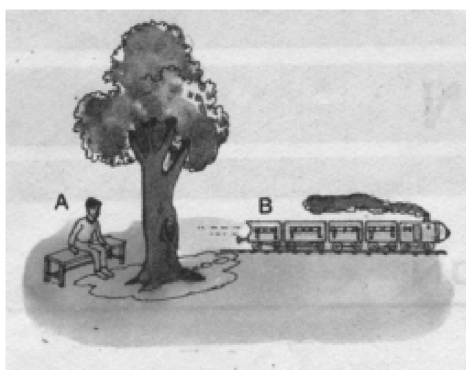
(ii) Revolution of earth around the sun for one complete revolution.

Rest :– A body is said to be at rest when its position does not change with time respect to the observer.

Motion :– A body is said to be in motion when its position changes with time respect to the observer.

Describe motion :

When a tree, is observed by an observer A sitting on a bench, the tree is at rest. This is because position of the tree is not changing with respect to the observer A.



Now, When the same tree T is observed by an observer sitting in a superfast train moving with a velocity v, then the tree is moving with respect to the observer because the position of tree is changing with respect to the observer B.

Rest and motion are relative terms : There is nothing like absolute rest. This means that an object can be at rest and also in motion at the same time i.e. all objects, which are stationary on earth, are said to be at rest with respect to each other, but with respect to the sun are making revolutions at 30 kmh^{-1}. In order to study motion, therefore, we have to choose a fixed position or point with respect to which the motion has to be studied. Such a point or fixed position is called a **reference point** or the **origin**. In order to describe the motion of an object we need to keep in mind three things;

(i) The distance of the body from a reference point. This reference point is called the origin of the motion of the body.

(ii) The direction of motion of the body.

(iii) The time of motion.

DO YOU KNOW ?

- Motion is an integral part of this universe. Everybody around us is in motion. From the motion of the electrons in their orbitals around the nucleus to the motion of the centre universe around its centre.

- Plants also show motion. Plants move towards stimulus. Plants also move when they grow in size. But since these movements are very slow, These are hardly perceived for a small interval of time.

- Mechanics deals with the motion of nonliving objects.

- Kinematics deals with the motion of non-living objects without taking into account the cause of their motion.

- Motion along a straight line is called translatory or rectilinear motion

- A point object is one whose dimensioons are very small as compared to the distance it travels.

- Rest and motion are relative terms. A body can be at rest as well as in motion at the same time.

SCALAR AND VECTOR QUANTITIES

Scalar Quantity :– A quantity that has only magnitude no direction is called a scalar quantity.

Ex. mass, time, distance, speed, work, power, energy, charge, area, volume, density, pressure, potential, temperature etc.

Vector Quantity :– The physical quantity that has magnitude as well as direction are called vector quantity.

Ex. velocity, acceleration, force, displacement, momentum, weight, electric field etc.

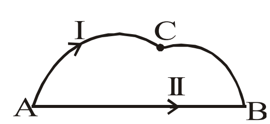
DIFFERECE BETWEEN SCALAR & VECTOR QUANTITIES :

	Scalar quantities	Vector quantities
1	These are completely specified by their magnitude only.	These are completely specified by their magnitude as well as direction.
2	These change by change of their magnitude only	These change by change of either their magnitude or direction or both
3	These are added or subtracted by laws of ordinary algebra like 4m+5m=9m.	These are added or subtracted by laws of vector addition.

DISTANCE AND DISPLACEMENT

Distance : The length of the actual path between the initial and the final position of a moving object in the given time interval is known as the distance travelled by the object.

Distance = Length of path I (ACB)



Distance is a scalar quantity.

Unit In SI system : metre (m)

In CGS system : centimetre (cm)

Large unit Kilometre (km)

Displacement:- The shortest distance between the initial position and the final position of a moving object in the given interval of time from initial to the final position of the object is known as the displacement of the object.

Displacement of an object may also be defined as the change in position of the object in a particular direction. That is,

Displacement of an object = Final position – Initial position of the object.

Displacement of an object may be zero but the distance travelled by the object in never zero.

Distance travelled by an object is either equal or greater than the magnitude of displacement of the object.

Displacement = Length of path II (AB) A to B,

displacement is vector quantities.

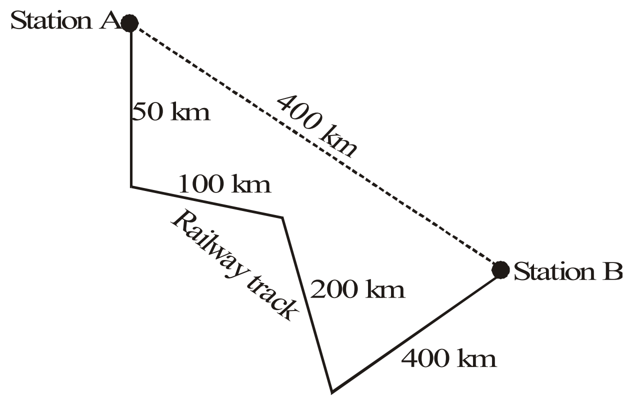
Units In SI system : metre (m)

In CGS system : centimetre (cm)

Ex. A train goes from station A to station B as shown in figure. Calculate

(i) the distance travelled by the train and

(ii) the magnitude of the displacement of the train on reaching station B.



Sol. (i) Distance travelled by the train = 50 + 100 + 200 + 400 = 750 km.

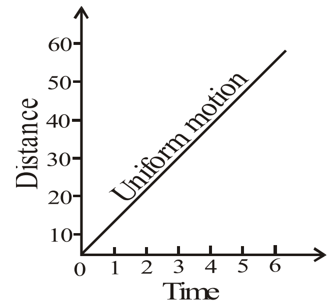
(ii) Magnitude of the displacement in going from station A to station B = 400 km.

DIFFERENCES BETWEEN DISTANCE AND DISPLACEMENT		
	Distance	**Displacement**
1	It is defined as the actual path traversed by a body.	It is the shortest distance between two points which the body moves.
2	It is a scalar quantity	It is a vector quantity
3	It can never be negative or zero	It can be negative, zero or positive.
4	Distance can be equal to or greater than displacement	Displacement can be equal to or less than distance.
5	Distance travelled is not a unique path between two points.	Displacement is a unique path between two points.
6	The distance between two points gives full information of the type of path followed by the body.	Displacement between two points does not give full information of the type of path followed by the body.
7	Distance never decreases with time. for a moving body it is never zero.	Displacement can decrease with time for a moving body it canbe zero.
8	Distance in SI is measured in metre	Displacement in SI is measured in metre.

Distance $\geq$ |displacement|

UNIFORM AND NON-UNIFORM MOTION

A moving body may cover equal distances in equal intervals of time or different distances in equal intervals of time. On the basis of above assumption, the motion of a body can be classified as uniform motion and non-uniform motion.



Uniform motion:

Time (in second)	0	1	2	3	4	5	6
Distance covered (in metre)	0	10	20	30	40	50	60

When a body covers equal distances in equal intervals of time however small may be time intervals, the body is said to describe a uniform motion.

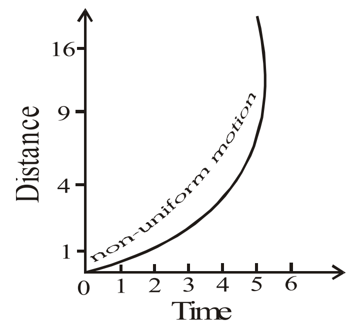
Example of uniform motion –

(i) An aeroplane flying at a speed of 600 km/h

(ii) A train running at a speed of 120 km/h

(iii) Light energy travelling at a speed of 3×10^8 m/s

(iv) A spaceship moving at a speed of 100 km/s



Non-uniform motion:

Time (in second)	0	1	2	3	4
Distance (in metre)	0	1	4	9	16

When a body covers unequal distances in equal intervals of time, the body is said to be moving with a non-uniform motion.

Example of non-uniform motion –

(i) An aeroplane running on a runway before taking off.

(ii) A freely falling stone under the action of gravity.

(iii) An object thrown vertically upward.

(iv) When the brakes are applied to a moving car.

QUESTIONS WITH SOLUTION

1. An object has moved through a distance. Can it have zero displacement ? If yes, support you answer with an example.

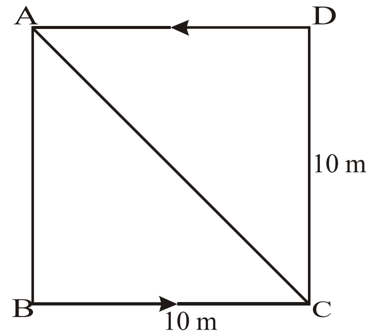
Ans. Yes, an object which has moved through a distance can have zero displacement.

Example : When a person, walking along a circular path, returns back to the starting point, after completing a circle, his displacement is zero. But he covers a distance $2\pi r$ where 'r' is the radius of circular path.

The displacement is zero, as the shortest distance between the initial and final position of the person is zero.

2. A farmer moves along the boundary of a square field of side 10 m in 40 s. What will be the magnitude of displacement of the farmer at the end of 2 minutes 20 seconds ?

Ans. The perimeter square field ABCD = 4 × 10m = 40 m.



Time for moving around the 10 m square field once = 40 s.

Time for journey of farmer = 2 min and 20 s = 140 s.

Number of times the farmer moves around the square field = $\frac{140}{40} = 3.5$ times.

For going once around the square field, the displacement = 0

For going thrice around the square field, the displacement = 0

For going $\frac{1}{2}$ times the square field, the distance covered = 40 m × $\frac{1}{2}$ = 20 m.

It is obvious from the figure, that if the farmer starts from pt A, then he will cover 10 m along AB and then 10 m along BC.

Therefore displacement of farmer from the point A to point C is

$$AC = \sqrt{(AB)^2 + (BC)^2} = \sqrt{(10)^2 + (10)^2} = 14.14 \text{ m}$$

3. Which of the following is true for displacement ?

(a) It cannot be zero.

(b) Its magnitude is greater than the distance travelled by the object.

Ans. None of the statement (a) or (b) is true for displacement.

4. Distinguish between speed and velocity.

Ans. (i) Speed is the rate of change of motion but velocity is the rate of change of motion in a specified direction.

(ii) Speed is a scalar quantity, but velocity is a vector quantity.

5. Under what condition is the magnitude of average velocity of an object equal to its average speed?

Ans. The magnitude of average velocity of an object is equal to its average speed when the velocity of an object changes at uniform rate, i.e., the body is in uniformly accelerated motion. If a body is moving with uniform acceleration.

Initial velocity = u, Final velocity = v

Average speed = Average velocity = $\frac{u + v}{2}$.

SPEED

Speed of a body is the distance travelled by the body per unit time. or The rate of change of motion is called speed.

$$\text{Speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

If a body covers a distance S in time t then speed,

$$v = \frac{S}{t}$$

Unit: In SI system : m/s or ms^{-1}

In CGS system : cm/s or cms^{-1}

Other km/h or kmh^{-1}

Important note : While comparing the speed of different bodies we must convert all speeds into same units. Speed is a scalar quantity, because it has the magnitude but no direction.

Uniform speed :- When a body covers equal distance in equal intervals of time, the body is to be moving with a uniform speed or constant speed.

Ex. (i) A train running with a speed of 120 km/h

(ii) An aeroplane flying with a speed of 600 km/h

Non-uniform speed :- When a body covers unequal distances in equal intervals of time, the body is said to be moving with non-uniform speed or variable speed.

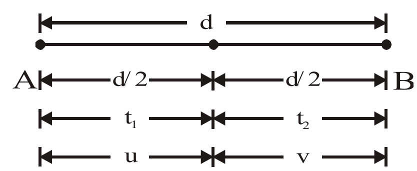
Ex. (i) A car running on busy road.

(ii) An aeroplane landing on runway.

Average speed :- The average speed of the body in a given time interval is defined as the total distance travelled, divided by the time interval.

$$\boxed{\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}}$$

Ex. A car travels first half distance with a uniform u and next half distance travels with a uniform speed v. Find its average speed.



Sol. Total distances $= \frac{d}{2} + \frac{d}{2} = d$

Total time $= t_1 + t_2 = t$

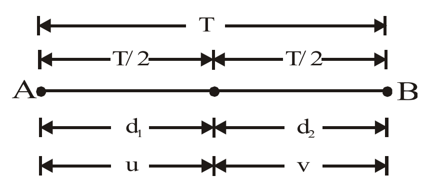
$\therefore \quad t_1 = \frac{d/2}{u}$ ...(i)

$t_2 = \frac{d/2}{v}$ ...(ii)

$V_{av} = \frac{d}{t}$ Putting the value of equation (i) and (ii)

$$V_{av} = \frac{d}{\frac{d/2}{u} + \frac{d/2}{v}} = \frac{d}{\frac{d}{2}\left(\frac{1}{u} + \frac{1}{v}\right)} = \frac{2}{\frac{v+u}{uv}} \qquad \boxed{V_{av} = \frac{2uv}{u+v}}$$

Ex. A car travels first half time with a uniform speed u and next half time with a uniform speed v. Find its average speed.



Sol. $\begin{pmatrix} \because d_1 = u \times \frac{T}{2} \ldots\text{(i)} \\ \because d_2 = v \times \frac{T}{2} \ldots\text{(ii)} \end{pmatrix}$

Total distances $d = d_1 = u \times \frac{T}{2} + v \times \frac{T}{2}$

$d = \frac{uT}{2} + \frac{vT}{2}$

Total time = T

$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time taken}}$

$$V_{av} = \frac{\frac{uT}{2} + \frac{vT}{2}}{T} = \frac{\frac{T}{2}(u+v)}{T} \qquad \boxed{V_{av} = \frac{u+v}{2}}$$

INSTANTANEOUS SPEED

The speed of a body at any particular instant of time during its motion is called the instantaneous speed of the body.

It is measured by **speedometer** in vehicles.

COMPETITIVE WINDOW

Comment : It can be very easily argued that

(a) The relative speed between two bodies A and B moving in the same direction with speed $|V_A|$ and $|V_B|$ i.e.

$|V_{same}|$ = difference in the speeds of two bodies A and B = $|V_A| - |V_B|$ or $|V_B| - |V_A|$ ... (A)

depending upon the fact whether $|V_A| > |V_B|$ or $|V_B| > |V_A|$

(b) The relative speed between the two bodies A and B moving in the opposite direction with speed $|V_A|$ and $|V_B|$ i.e.

$|V_{opposite}|$ = sum of the speed of the two bodies A and B = $|V_A| + |V_B|$ ... (B)

It should be carefully noted that equation A and B are valid only for one-dimensional motion and not in two and three dimension motion.

VELOCITY

The velocity of a body is the displacement of a body per unit time.

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time}}$$

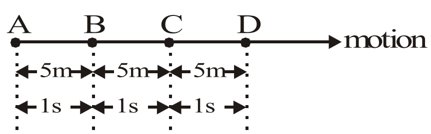
The displacement covered by a body per unit time or the speed of a body in specified direction is called the velocity.

Unit: In SI system : m/s or ms^{-1}

In CGS system : cm/s or cms^{-1}

Other km/h or kmh^{-1}, km/min.

Uniform velocity



Body moving with uniform velocity

When a body covers equal displacement in equal interval of time, the body is said to be moving with a uniform velocity.

Conditions for uniform velocity :-

(i) The body must cover equal displacement in equal intervals of time.

(ii) The direction of motion of the body should not change.

Ex. (i) A train running towards south with a speed of 120 km/h.

(ii) A aeroplane flying due north-east with a speed of 600 km/h.

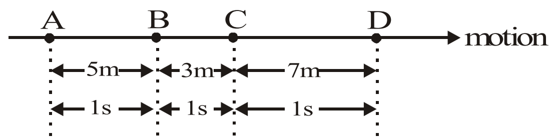
Very important note :-

Direction of velocity represent direction of motion of body.

O R

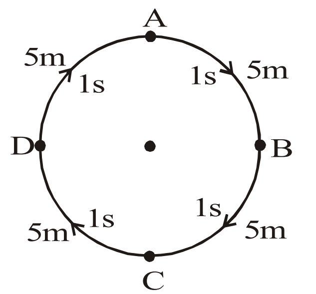
Sign of velocity represent the direction of motion of body.

Non-uniform velocity/variable velocity :



Body moving with non-uniform velocity

When a body covers unequal displacement in equal intervals of time, the body is said to be moving with variable velocity.



Body moving with variable velocity

When a body covers equal distance in equal intervals of time, but its direction changes, then the body is said to be moving with variable velocity.

Conditions for variable velocity :-

- It should cover unequal displacement in equal intervals of time.
- It should cover equal distances in equal intervals of time but its direction must change.

Ex.
- A car running towards north on a busy road has a variable velocity as the displacement covered by it per unit time changes with change in the road condition.
- The blades of a rotating ceiling fan, a person running around a circular track with constant speed etc. are the example of variable velocity, as the direction of the moving body changes in each case.

Average velocity :

Total displacement divided by total time is called an average velocity.

$$\boxed{\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time taken}}}$$

$$V_{av} = \frac{x_2 - x_1}{t_2 - t_1}$$

O R

The arithmetic mean of initial velocity and final velocity for a given time period, is called average velocity.

$$\boxed{\text{Average velocity} = \frac{\text{Initial velocity + Final velocity}}{2}}$$

$$v_{av} = \frac{u + v}{2}$$ where u = initial velocity, v = final velocity

Memorise : When a body moves with constnat velocity, the average velocity is equal to instantaneous velocity. The body is said to be in uniform motion.

DIFFERENCE BETWEEN SPEED AND VELOCITY		
	Speed	**Velocity**
1	It is defined as the rate of change of distance	It is the shortest distance between two points between which the body moves.
2	It is a scalar quantity	It is a vector quantity
3	It can never be negative or zero	It can be negative, zero or positive
4	Speed is velocity without direction.	Velocity is directed speed
5	Speed may or may not be equal to velocity.	A body mass possess different velocities but the same speed.
6	Speed never decreases with time. For a moving body it is never zero.	Velocity can decrease with time. For a moving body it can be zero.
7	Speed in SI is measured in ms^{-1}	Velocity in SI, is measured in ms^{-1}.

ACCELERATION

The rate of change of velocity of a moving body with time is called acceleration.

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time taken for change}}$$

but change in velocity = final velocity – initial velocity.

$$\text{Acceleration} = \frac{\text{Final velocity} - \text{Inital velocity}}{\text{Time taken for change}}$$

$$\vec{a} = \frac{\vec{v} - \vec{u}}{t}$$

If body moves with uniform velocity, then v = u and then acceleration is zero i.e. a = o.

Unit of accelration

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time}},\ \text{Acceleration} = \frac{\text{Unit of velocity}}{\text{Unit of time}},\ \text{Acceleration} = \frac{m/s}{s} = m/s^2$$

In SI system is m/s^2 or ms^{-2}

In CGS system is cm/s^2 or cms^{-2}

Positive Acceleration : *If the velocity of an object increases with time in the direction of the motion of the object, then the acceleratin of the body is known as positive acceleration.*

In this case, the object **pick up the speed** in a particular direction (i.e., velocity). For example, if an object starts from rest and its velocity goes on increasing with time in the direction of its motion, then the object has positive acceleration. The direction of positive acceleration is in the direction of motion of the object.

Negative Acceleration : *If the velocity of an object decreases with time, then the acceleration of the object is known as negative acceleration.*

It is written as $-\vec{a}$

For example, if an object moving with certain velocity is brought to rest then the object is said to have negative acceleration.

Acceleration without changing speed :

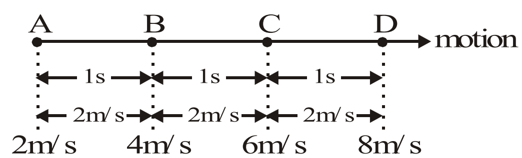
When an object moves in a circular path with constant speed, then its velocity changes due to the change in the direction of motion of the object. hence, the object is accelerated without changing its speed.

In this case, the direction of acceleration is towards the centre of the circular path.

DO YOU KNOW ?

- *Positive or negative sign of acceleration always shows the direction of acceleration or direction of force but not represent direction of motion of body.*
- *Acceleration which oppose the motion of a body is called retardation or negative acceleration.*
- *If sign of velocity and acceleration are same it means speed of body will always increase.*
- *If both are opposite sign it means speed of body will always decrease.*

Uniform acceleration :



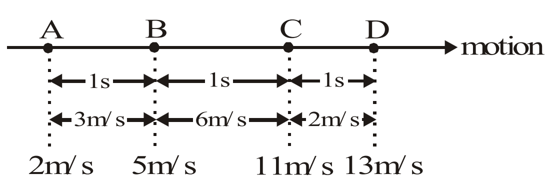
Uniformly accelerating body

When a body undergoes equal changes in velocity in equal intervals of time, the body is said to be moving with a uniform acceleration

Ex. (i) Motion of a freely falling body.

(ii) Motion of a ball rolling down on an inclined plane.

Non-uniform acceleration or variable acceleration :



non-uniform accelerating body

When a body describes unequal change in velocity in equal intervals of time, the body is said to be moving with non-uniform acceleration.s

Ex. (i) The motion of a bus leaving or entering the bus stop.

(ii) The motion of a train leaving or entering the platform.

(iii) A car moving on a busy road has non-uniform acceleration.

EQUATIONS OF UNIFORMLY ACCELERATED MOTION

These equations give relationship between initial velocity, final velocity, time taken, acceleration and distnace travelled by the bodies.

FIRST EQUATION OF MOTION :

A body having an initial velocity 'u' acted upon by a uniform acceleration 'a' for time 't' such that final velocity of the body is 'v'.

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time taken}} \qquad \text{Acceleration} = \frac{\text{Final velocity} - \text{Initial velocity}}{\text{Time taken}}$$

$$a = \frac{v-u}{t} \qquad v - u = at \qquad \boxed{v = u + at}$$

Where ; v = final velocity of the body u = initial velocity of the body

a = Acceleration t = time taken

SECOND EQUATION OF MOTION :

It gives the distance travelled by a body in time t.

A body having an initial velocity 'u' acted upon by a uniform acceleration 'a' for time 't' such that final velocity of the body is 'v' and the distance covered is 's'.

The average velocity is given by :-

$$\text{Average velocity} = \frac{\text{Initial velocity} + \text{Final velocity}}{2} \qquad V_{av} = \frac{u+v}{2}$$

$$\text{distance covered} = \text{average velocity} \times \text{time} \qquad s = \left(\frac{v+u}{2}\right) \times t$$

but v = u + at (from first equation of motion)

Thus, $$s = \left(\frac{u+at+u}{2}\right) \times t = \left(\frac{2u}{2} + \frac{at}{2}\right) \times t$$

$$\boxed{s = ut + \frac{1}{2}at^2}$$

Where ; s = distance travelled by the body in time t

u = initial velocity of the body

a = Acceleration

t = time taken

THIRD EQUATION OF MOTION :

A body having an initial velocity 'u' moving with a uniform acceleration 'a' for time 't' such that final velocity 'v' and the distance covered is 's'. the third equation of motion is $v^2 = u^2 + 2$ at. it gives the velocity acquired by a body in travelling a distance s.

$$v = u + at \qquad \text{.....(i)}$$

$$s = ut + \frac{1}{2}at^2 \qquad \text{.....(ii)}$$

Squaring eq. (i), we have

$$v^2 = (u + at)^2$$

$$v^2 = u^2 + 2uat + a^2t^2$$

$$v^2 = u^2 + 2a\left(ut + \frac{1}{2}at^2\right) \qquad \text{.....(iii)}$$

Substituting the value of eq (ii) in eq. (iii), we get. $\boxed{v^2 = u^2 + 2as}$

QUESTIONS WITH SOLUTION

6. What does speedometer of an automobile measure ?

Sol. The speedometer measures the instantaneous speed of the automobile at some particular time.

7. What does the path of an object look like when it is in uniform motion ?

Sol. The path of an object will be a straight line.

8. During an experiment, a signal from a spaceship reached the ground station in five minutes. What was the distance of the spaceship from the ground station ? The signal travels at the speed of light, that is 3×10^8 ms^{-1}.

Sol. Speed of signal = 3×10^8 ms^{-1}

Time in which signal reaches ground = 5 min = 5 × 60 = 300 s

Distance of spaceship from the ground level = speed × time = $3 \times 10^8 \times 300 = 9 \times 10^{10}$ m

9. When will you say a body is in

(i) Uniform acceleration.

(ii) Non uniform acceleration ?

Sol. (i) A body is in uniform acceleration when equal changes in velocity take place in equal intervals of time, however small these intervals may be.

(ii) A body is said to be possessing non-uniform acceleration when unequal changes in velocity take place in equal intervals of time, however small these intervals may be.

10. A bus decreases its speed from 80 km h^{-1} to 60 km h^{-1} in 5 s. Find the acceleration of the bus.

Sol. Given t = 5 s

Initial speed of bus

$$u = 80 \text{ km h}^{-1} = 80 \times \frac{5}{18} = 22.2 \text{ ms}^{-1}$$

Final speed of the bus

$$v = 60 \text{ km h}^{-1} = 60 \times \frac{5}{18} = 16.7 \text{ ms}^{-1}$$

Now acceleration is given by the relation

$$a = \frac{v-u}{t} = \frac{16.7 - 22.2}{5} = -1.1 \text{ ms}^{-1}$$

11. A train starting from a railway station and moving with uniform acceleration attains a speed 40 kmh^{-1} in 10 minutes. Find its acceleration.

Sol. Given t = 10 min = 10 × 60 = 600 s

Initial speed of train, u = 0 ms^{-1}

Final speed of train

$$v = 40 \text{ km h}^{-1} = 40 \times \frac{5}{18} = 11.1 \text{ ms}^{-1}$$

Now acceleration is given by the relation

$$a = \frac{v-u}{t} = \frac{11.1 - 0}{600} = 0.0185 \text{ ms}^{-1}$$

12. What is the nature of the distance time graphs for uniform and non-uniform motion of an object ?

Sol. The distance time-graph for uniform motion is a straight line not parallel to the time axis. The distance time-graph for non-uniform motion is not a straight line. It can be a curve or a zigzag line not parallel to time axis.

13. What can you say about the motion of an object whose distance time-graph is a straight line parallel to the time axis ?

Sol. The object is stationary.

14. What can you say about the motion of an object if its speed-time graph is a straight line parallel to the time axis ?

Sol. The object has uniform motion.

15. What is the quantity which is measured by the area occupied below the velocity-time graph ?

Sol. Displacement is the quantity which is measured by the area under velocity time graph.

16. A bus starting from rest moves with a uniform acceleration of 0.1 ms^{-2} for 2 minutes. Find

(a) the speed acquired.

(b) the distance travelled.

Sol. Given

Initial speed of bus, $u = 0\ ms^{-1}$

Final speed of bus, $v = ?$

$a = 0.1\ ms^{-2}$, $t = 2\ min = 120\ s$

$S = ?$

(i) We know, $v = u + at$

or $v = 0 + 0.1 \times 120 = 12\ ms^{-1}$

(ii) $S = ut + \frac{1}{2}at^2$

$$S = 0 \times 120 + \frac{1}{2} \times 0.1 \times (120)^2 = 720\ m$$

Therefore

Final speed acquired = $12\ ms^{-1}$

Distance travelled = 720 m

DO YOU KNOW ?

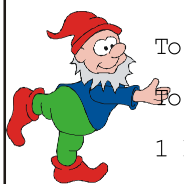
IMPORTANT NOTE:-

- *Velocity in m/s* $= \frac{5}{18} \times$ *velocity in km/h.*

  $36 \text{ km h}^{-1} = 36000 \text{ m h}^{-1}$

  $$= \frac{36000}{60 \times 60} \text{ ms}^{-1} = 10 \text{ ms}^{-1}$$

  $1 \text{ km h}^{-1} = \frac{10}{36} \text{ms}^{-1}$ or $1 \text{ km h}^{-1} = \frac{5}{18} \text{ms}^{-1}$



  To convert km h^{-1} to ms^{-1}, multiply by 5/18.

  To convert ms^{-1} to km h^{-1}, multiply by 18/5.

  1 km = 1000 m

  1m = 100 cm = 1000 mm

- *Distance in kilometres should be converted into metre.*
- *Before solving problems, asure that the data provided have the same system of unit, i.e. either they should be in SI system or CGS system.*
- *If a body start from rest, its initial velocity (u) is zero, (u =0)*
- *If a moving body comes to rest/stops, its final velocity (v) is zero, (v =0)*
- *If a body is moving with uniform velocity, its acceleration is zero, (a =0)*

GRAPHICAL REPRESENTATION OF MOTION

Graph :

A graph is a line, straight line or curved, showing the relation between two variable quantities of which one varies as a result of the change in the other.

The quantity which changes independently is called independent variable and the one which changes as a result of the change in the other is called dependent variable.

Plotting a graph :

- Take independent varibale along X-axis and dependent variable along Y-axis.
- Choose convenient scale so that more than 2/3rd of graph is filled.
- Draw free hand curve to join them.

Uses of graph :

- It gives a bird's eye view of the changes.
- It is used to show dependence of one quantity on the other e.g., distance or velocity on time.
- We can find distance covered in a given time.
- Slope of velocity-time graph gives acceleration.
- We can find position or velocity of body at any instant.

COMPITITIVE WINDOW

- *A graph gives not only the relation between two variable quantities in a pictorial form but also enable us to study of nature of motion.*
- *Speed = slope of distance time graph.*
- *If distance-time graph is parallel to time axis, then the body is stationary.*
- *If distance-time graph is a straight line having constant gradient or slope, then the body has uniform motion.*
- *If distance-time graph is a curve with increasing gradient or slope, then the body has non-uniform motion*
- *If slope of distance-time graph increases, the speed of the body increases.*
- *If slope of distance-time graph of non-uniform motion decreases, the speed of the body decreases.*

Distance/Displacement – Time graph :

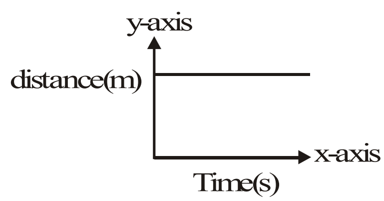
This graph is plotted between the time taken and the distance covered, the time is taken along the x-axis and the distance covered is taken along the y-axis.

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

- The slope of the distance-time graph gives the speed of the body.
- The slope of the displacement-time graph gives the velocity of the body.

When the body is at rest :

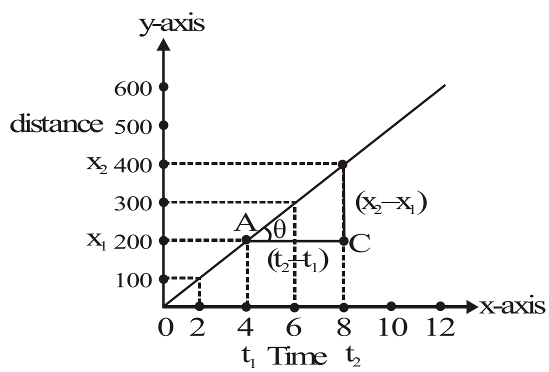
When position of the body does not change with time then it is said to be stationary, the distance-time graph of such a body is a straight line parallel to x-axis.



distance-time graph for a stationary body.

When the body is in uniform speed:

When the position of the body changes by equal intervals of time then body is said to be moving with uniform speed. The distance-time graph of such a body is a straight line, inclined to x-axis.

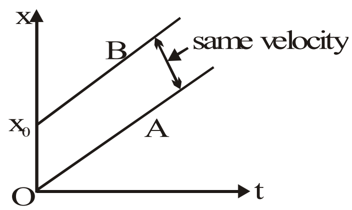


$$\text{Slope} = \frac{\text{measure on y-axis}}{\text{measure on x-axis}}$$

$$\text{Slope} = \frac{x_2 - x_1}{t_2 - t_1}$$

$\frac{\text{displacement}}{\text{time}}$ = velocity $V = \frac{\Delta x}{\Delta t}$. **O R** $\frac{\text{distance}}{\text{time}}$ = speed

Special case-I

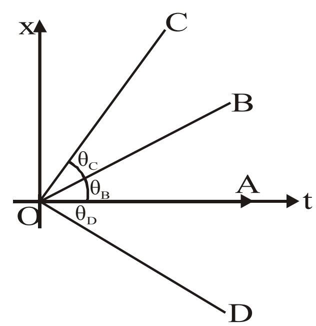


In uniform motion along a straight line the position x of the body at any time t is related to the constant velocity as,

$x_A = vt$ Starting form zero

$x_B = x_0 + vt$ starting from x_0

Special case-II



Slope of line A = $\tan\theta_A = \tan 0$ $(\because \theta_A = 0)$

= zero velocity

Slope of line B = $\tan\theta_B$ = positive velocity

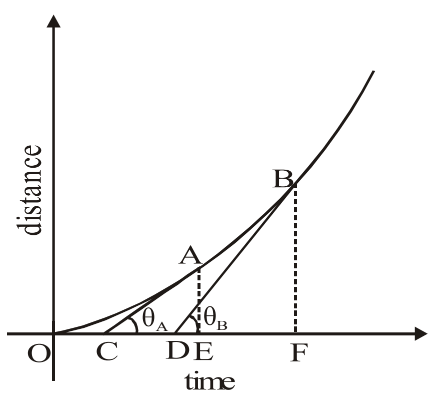
Slope of line C = $\tan\theta_C$ = more positive velocity

$\because \theta_C > \theta_B$ $(\tan\theta_C > \theta_B)$

Then $v_C > v_B$

Slope of line D = $\tan(-\theta_D)$ = negative velocity.

When the body is in motion with a non-uniform (variable) speed.



distance-time graph for a body moving with non-unifrom speed.

The position-time graph is not a straight line, but is a curve.

The speed of the body at any point is known as instantaneous speed and can be calculated by finding the slope at that point.

So instantaneous speed of the body at point A.

Slope at point A = $\tan\theta_A = \frac{AE}{CE}$

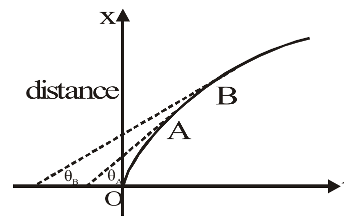
instantaneous speed of the body at point B

Slope at point B = $\tan\theta_B = \frac{BF}{DF}$

$\theta_B > \theta_A$ so slope at point B is greater than the slope at point A.

Hence speed of body at point B is a greater than, the speed of body at point A.

When the speed decreases with passage of time –

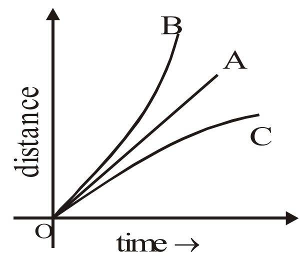


Slope at point A > slope at point B ($\therefore \theta_A > \theta_B$)

So, speed at point A > speed at point B

> ***Important note :–*** *A distance time graph can never be parallel to y-axis (representing distance) because this line has slope of 90° and slope = tanθ = tan90° = infinite, which means infinite speed. It is impossible.*

Acceleration from displacement-time graph.



For line A :– A straight line displacement-time graph represents a uniform velocity and zero acceleration

for line B :– A curved displacement-time graph rising upward represents an increasing velocity and positive acceleration

For line C :– A curved displacement-time graph falling downwards, represents a decreasing velocity and negative acceleration.

VELOCITY-TIME GRAPH :

The variation in velocity with time for an object moving in a straight line can be represented by a velocity-time graph. In this graph, time is represented along the x-axis and velocity is represented along the y-axis.

$$\text{Acceleration} = \frac{\text{speed or velocity}}{\text{time}}$$

hence the slope of the speed/velocity-time graph, gives the acceleration of the body.

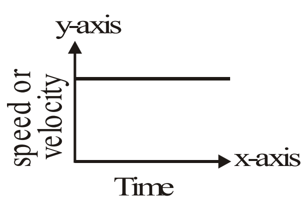
Distance = speed × time

hence, area enclosed between the speed-time graph line and x-axis (time axis) gives the distance covered by the body. Similarly area enclosed between the velocity-time graph line and the x-axis (time axis) gives the displacement of the body.

Note: Since the graph takes into account, only the magnitude hence velocity-time graph is not different from speed-time graph.

When the body is moving with constant velocity :

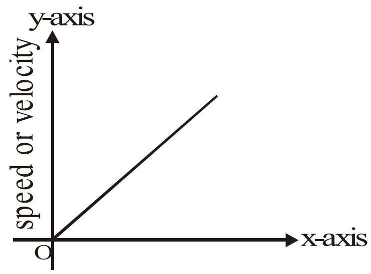
When the body moves with constant velocity i.e. its motion is uniform.



The speed or velocity of the body is uniform hence the magnitude remains same. The graph is a straight line parallel to x-axis (time-axis). Since the velocity is uniform. Its acceleration is zero. The slope of the graph in this case is zero.

CONCLUSION : Velocity-time graph of a body moving with constant velocity is a straight line parallel to time axis.

When the body is moving with a uniform acceleration.

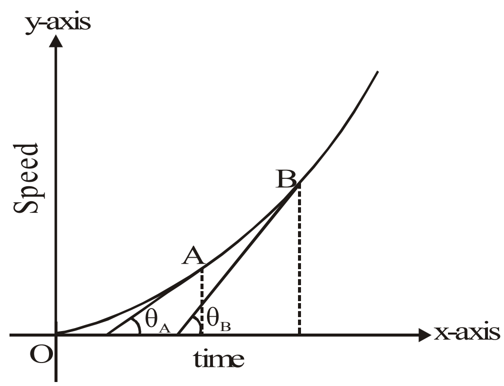


Time

The speed or velocity is changing by equal amounts in equal interval of time, the speed or velocity time graph of such a body is a straight line inclined to x-axis (time-axis).

When the body is moving with a non-uniform (variable) acceleration.

the speed or velocity-time graph is not a straight line but is a curve.



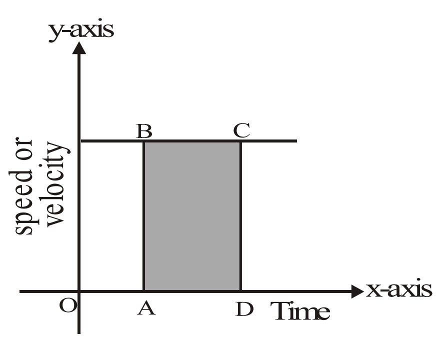
The line has different slopes at different times, its acceleration is variable. At point A, slope is less hence acceleratoin is less. At point B slope is more hence acceleration is more.

Note: *Speed or velocity-time graph line can never be paralled to y-axis (speed axis), because slope angle becomes 90° than tan90° is infinite it is impossible.*

Distance or displacement from speed or velocity-time graph.

As distance or displacement = speed or velocity x time, hence the distance or displacement can be calculated from speed or velocity-time graph.

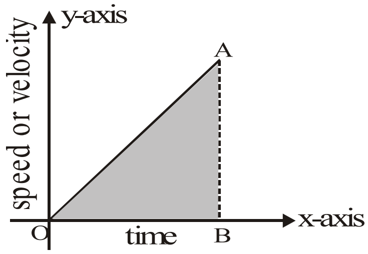
When speed or velocity is uniform (constant)



Distance/displacement = Area of rectangle ABCD = AB × AD

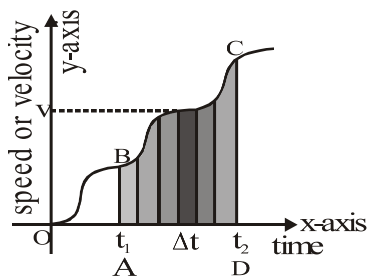
Thus, We find that the area enclosed by velocity-time graph and the time axis gives the distance travelled by the body.

When acceleration is uniform (constant)



distance or displacement = area of right triangle OAB = $\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times OB \times BA$

When speed or velocity as well as acceleration is non-uniform (variable).



The speed-time graph of a body moving irregularly with variable speed and acceleration. For a small interval of time Δt, as there is not much change in speed, hence the speed can be taken as constant.

$\therefore$ For this small time interval.

Distance $\Delta s = v\Delta t$ = Area of the blackened strip.

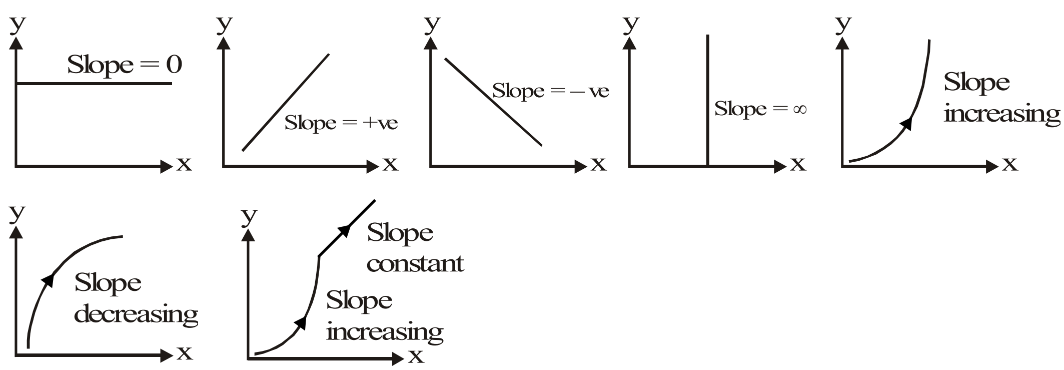
For whole time interval between t_1 and t_2, distance = sum of areas of all the strips, between t_1 and t_2 = area of shaded figure ABCD.

Application of Velocity- time Graphs :

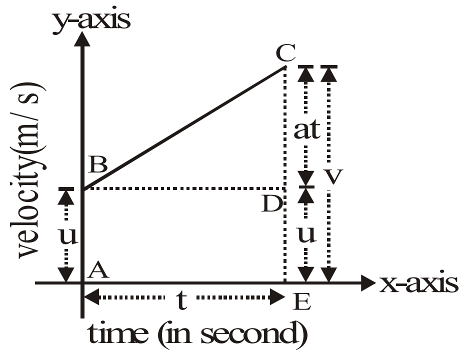
A number of useful results can be deduced from velocity time graph.

- Slope of velocity-time graph gives the acceleration.
- Area below velocity-time graph and the time axis gives the distance covered.
- Using the above two results, we can derive all equations of motion.

Natre of slope :- **COMPETITIVE WINDOW**



GRAPHICAL REPRESANTATION OF EQUATION OF MOTION :



Represents a velocity-time graph BC, in which AB represents the initial velocity u, CE represents final velocity v, such that the change in velocity is represented by CD, which takes place in time t, represented by AE.

Derivation of v = u + at

Acceleration = slope of the graph line BC.

$$a = \frac{CD}{BD} = \frac{CE - DE}{BD} \quad \text{OR} \quad a = \frac{v-u}{t}$$

$$\left[\begin{array}{l} \because DE = AB = u \\ \because BD = AE = t \\ \quad CE = V \end{array}\right]$$

$v - u = at$

$\boxed{v = u + at}$

Derivation of $s = ut + \frac{1}{2}at^2$

Distance travelled = Area of trapezium ABCE

= Area of rectangle ABDE + Area of triangle BCD

$= AB \times AE + \frac{1}{2}(BD \times CD) = t \times u + \frac{1}{2}[t \times (v-u)]$

$$\left[\begin{array}{l} \because BD = AE = t \\ \because CD = CE - DE = v - u \end{array}\right]$$

$\because \quad v = u + at = u \times t + \frac{1}{2}[t \times (u + at - u)]$

$\boxed{S = ut + \frac{1}{2}at^2}$

Derivation of $v^2 = u^2 + 2as$

From the velocity-time graph distance covered = Area of trapezium ABCE

$\Rightarrow \quad S = \frac{1}{2}(AB + CE) \times AE \qquad S = \frac{1}{2}(u + v) \times t \qquad$ ...(i)

$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time}}$

$a = \frac{v-u}{t} \qquad t = \frac{v-u}{a} \qquad$ ...(ii)

Substituting the value of t in eq (i)

$S = \frac{(v+u)}{2} \times \frac{(v-u)}{a} \qquad \because A^2 - B^2 = (A + B) \times (A - B)$

$S = \frac{v^2 - u^2}{2a}$

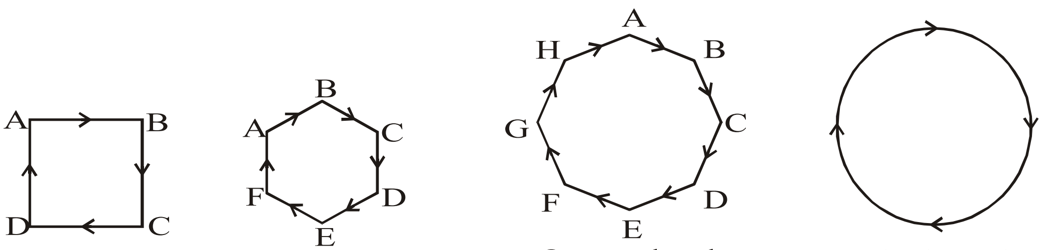
$v^2 - u^2 = 2as \quad \boxed{v^2 = u^2 + 2as}$

CIRCULAR MOTION

Motion of a particle (small body) along a circle (circular path), is called circular motion.

If the body covers equal distances along the circumference of the circle, in equal intervals of time, then motion is said to be a uniform circular motion. When a body moves along a circular path, then its direction of motion changes continuously.

Note:- *A uniform circular motion is a motion in which speed remain constant but direction of velocity changes continuously.*



square track four sides | Hexagonal track six sides | Octagonal track eigth sides | Circular track Indefinite number of sides

Examples of uniform circular motion are:-

(i) Motion of moon around the earth.

(ii) Motion of a satellite around its planet.

(iii) Motion of earth around the sun.

(iv) An athlete running on a circular track with constant speed.

(v) Motion of tips of the second hand, minute hand and hour hand of a wrist watch.

DO YOU KNOW?

Circular motion is an accelerated motion.

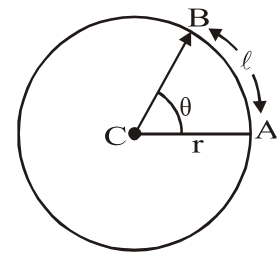
In a circular motion, velocity changes in direction only, the motion is said to be accelerated.

Uniform linear motion is not accelerated but uniform circular motion is accelerated.

Difference between a uniform linear motion and a uniform circular motion.

	Uniform linear motion	Uniform circular motion
1	The direction of motion does not change.	The direction of motion changes continuously.
2	The motion is non-accelerated	The motion is accelerated

RADIAN



It is a convenient unit for measuring angles in physics.

The arc AB of the circle, has length ℓ and subtends an angle θ at the centre C.

ff $\angle ACB = \theta$ radians

Then $\theta = \dfrac{\text{arc}}{\text{radius}}$ $\theta = \dfrac{\ell}{r}$

when $\ell = r$ then $\theta = 1$ radian.

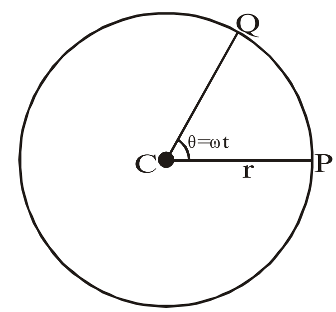
One radian is defined as the angle subtended at the centre of the circle by an arc which is equal in length to its radius.

Angle subtended by the circumference at the centre.

$\theta = \dfrac{2\pi r}{r} = 2\pi$ radians 2π radians $= 360°$ 1 radian $= \dfrac{360°}{2\pi}$

$\boxed{1 \text{ radian} = 57.3°}$

ANGULAR DISPLACEMENT AND ANGULAR VELOCITY :



The angle covered by a body in 1 sec. is called angular velocity.

It is usually denoted by w and measured in radian per sec.

If θ is the angle covered in time 't' then :

Angular velocity $= \dfrac{\text{Angular displacement}}{\text{Time taken}}$

$\omega = \dfrac{\theta}{t}$

$\boxed{\theta = \omega t}$

Unit Angular displacement $= \theta$ (in radian)

Angular velocity ω

$\omega = \dfrac{\theta}{t}$

$\Rightarrow \dfrac{\text{radian}}{\text{second}} \Rightarrow$ rad/s

Relation between linear velocity and angular velocity.

Linear displacement $= \ell$

Angular displacement $= \theta$

$\theta = \dfrac{\ell}{r}$

$\ell = r\theta$

for a time inervals t

Linear velocity $v = \dfrac{\ell}{t}$

Angular velocity $\omega = \dfrac{\theta}{t}$

$\omega = \dfrac{\ell}{r.t}$

$\omega = \dfrac{v}{r}$ $\boxed{v = r\omega}$

QUESTION WITH SOLUTION

17. A train is travelling at a speed of 90 kmh^{-1}. Brakes are applied so as to produce a uniform acceleration of – 0.5 ms^{-2}. Find how far the train will go before it is brought to rest.

Sol. Given

Initial speed of train,

$$u = 90 \text{ km h}^{-1} = 90 \times \frac{5}{18} = 25 \text{ ms}^{-1}$$

Final speed, $v = 0 \text{ ms}^{-1}$

Acceleration $a = -0.5 \text{ ms}^{-2}$

Distance covered, S = ?

Using the relation $v^2 - u^2 = 2aS$, we have

$$S + \frac{v^2 - u^2}{2a} = \frac{0 - (25)^2}{2 \times (-0.5)} = 625 \text{ m}$$

18. A trolley, while going down an inclined plane, has an acceleration of 2 cms^{-2}. What will be its velocity 3 s after the start ?

Sol. Given

Initial velocity, u = 0

Final velocity, v = ?

Time, t = 3 s

Acceleration, $a = 2 \text{ cms}^{-2}$

We know that v = u + at

Or $v = 0 + 2 \times 3 = 6 \text{ cms}^{-1}$

Therefore, final velocity = 6 cms^{-1}.

19. A racing car has uniform acceleration of 4 ms^{-2}. What distance will it cover in 10 s after start ?

Sol. Given

Initial velocity, u = 0

Acceleration, $a = 4 \text{ ms}^{-2}$

Time, t = 10 s

Distance covered, S = ?

We know ; $S = ut + \frac{1}{2}at^2$

$$S = 0 \times 10 + \frac{1}{2} \times 4 \times (10)^2$$

= 0 + 200 = 200 m

Therefore, distance covered = 200 m.

20. A stone is thrown in a vertically upward direction with a velocity of 5 ms^{-1}. If the acceleration of the stone during its motion is 10 ms^{-2} in the downward direction, what will be the height attained by the stone and how much time will it take to reach there ?

Sol. Given

Initial velocity, $u = 5\ ms^{-1}$

Final velocity, $v = 0$

Acceleration in the downward direction = 10 ms^{-2}

Therefore acceleration in the upward direction

$a = -10\ ms^{-2}$

Height attained by stone, S = ?

Time taken to attain height, t = ?

(i) Using the relation ; $v = u + at$

$0 = 5 + (-10)\ t$ or

$t = 5/10 = 0.5$ s

(ii) Using the relation ; $v^2 - u^2 = 2aS$, we have

$$S = \frac{v^2 - u^2}{2a} = \frac{(0)^2 - (5)^2}{2 \times (-10)} = 1.25\ m$$

NCERT QUESTION NO- 1 TO LAST

1. An athlete completes one round of a circular track of diameter 200 m in 40 s. What will be the distance covered and the displacement at the end of 2 minutes 20 s ?

Sol. Given

Diameter of circular track, 2r = 200 m

Circumference of circular track = $2\pi r$

$$S = \pi(2r) = \frac{22}{7} \times 200 = \frac{4400}{7}\ m$$

Time for completing one round = 40 s.
Time for which the athlete ran = 2 min and 20 s = 140 s
Now distance coverd by the athlete in 40 s.

$$S = \frac{4400}{7}$$

(i) Therefore, distance covered by athlete in 140 s.

$$= \frac{4400}{7} \times \frac{140}{40} = 2200\ m$$

(ii) As the athlete returns to the initial point in 40 s, this displacement = 0

Now

Number of rounds in 40 second = 1

Hence number of rounds in 140 s is = $\frac{140}{40}$ = 3.5.

For each complete round the displacement is zero.

Therefore for 3 complete rounds, the displacement will be zero.

The final displacement will be due to half the round.

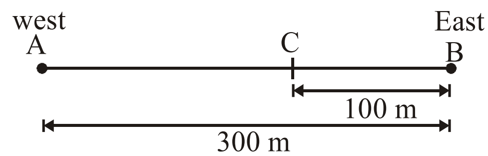
In half round distance covered = half of circumference.

Thus, his displacement = diameter of circular track = 200 m

Displacement after 140 s = 200 m

2. Joseph jogs from one end A to the other end B of a straight 300 m road in 2 minutes 50 seconds and then turns around and jogs 100 m back to point C in another 1 minute. What are Joseph's average speeds and velocities in Jogging (a) from A to B and (b) from A to C ?

Sol. The required figure is as shown



Distance covered = 300 m

Time taken = 2 min and 50 s = 170 s

Now average speed from A to B is given by

$$V_{av} = \frac{\text{distance covered}}{\text{time}} = \frac{300}{170} = 1.76\ \text{ms}^{?1}$$

Now average velocity from A to B is given by

$$V_{av} = \frac{\text{displacement}}{\text{time}} = \frac{300}{170} = 1.76\ \text{ms}^{?1}$$

When Joseph turns around from B to C toward west, then

Distance covered = 300 + 100 = 400 m and

Time taken = 170 + 60 = 230 s

Therefore average speed from A to C is

$$V_{av} = \frac{\text{distance covered}}{\text{time}} = \frac{400}{230} = 1.74\ \text{ms}^{-1}$$

Now displacement from A to C = 200 m

Therefore average velocity from A to C is

$$V_{av} = \frac{\text{displacement}}{\text{time}} = \frac{200}{230} = 0.869\ \text{ms}^{-1}$$

3. Abdul while driving to school computes the average speed for his trip to be 20 km h^{-1}. On his return trip along the same route, there is less traffic and the average speed is 40 km h^{-1}. What is the average speed for Abdul's trip ?

Sol. Let one way distance for his trip be S.

Let t_1 be the time for his trip from home to school and t_2 be the time for his return trip.

$$\text{Then } t_1 = \frac{S}{v_1} = \frac{S}{20}\text{h and } t_2 = \frac{S}{v_2} = \frac{S}{40}\text{h}$$

Therefore total time of trip is

$$T = t_1 + t_2 = \frac{S}{20} + \frac{S}{40} = \frac{3S}{40}\text{h}$$

Total distance covered = 2S

Therefore average speed of Abdul

$$V_{av} = \frac{\text{total distance}}{\text{total time}} = \frac{2S \times 40}{3S} = 26.7\ \text{kmh}^{-1}$$

4. **A motorboat starting from rest on a lake accelerates in a straight line at a constant rate of $3.0\ ms^{-2}$ for 8.0 s. How far does the boat travel during this time ?**

Sol. **Given :** Initial velocity of boat, u = 0

Acceleration, $a = 3.0\ m\ s^{-2}$

Time, t = 8 s

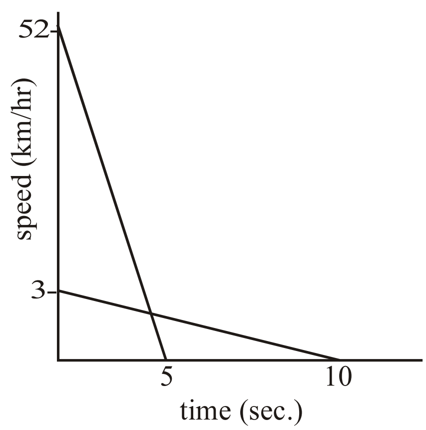
Distance covered, S = ?

Using the relation $S = ut + \frac{1}{2}at^2$ we have

$$S = 0 \times 8 + \frac{1}{2} \times 3 \times 8^2 = 96\ m.$$

5. **A driver of a car travelling at $52\ km\ h^{-1}$ applies the brakes and accelerates uniformly in the opposite direction. The car stops in 5s. Another driver going at $3\ kmh^{-1}$ in another car applies his brakes slowly and stops in 10 s. On the same graph paper, plot the speed versus time graphs for the two cars. Which of the two cars travelled farther after the brakes were applied ?**

Sol. The speed time graph of both the cars are shown below.



(i) Distance covered by car moving at $52\ kmh^{-1}$

or $52 \times \frac{5}{18} = 14.4\ ms^{-1}$.

$= \text{area of } \Delta POQ = \frac{1}{2} \times PQ \times QR = \frac{1}{2} \times 14.4 \times 5 = 36\ m$

(ii) Distance covered by car moving at $3\ kmh^{-1}$

or $3 \times \frac{5}{18} = 0.83\ ms^{-1}$

$\text{area of } \Delta OLN = \frac{1}{2} \times LO \times ON = \frac{1}{2} \times 0.83 \times 10 = 4.15\ m$

The car moving at $52\ km\ h^{-1}$ travels more distance on the application of brakes.

6. **Figure below shows the distance-time graph of three object A, B and C. Study the graph and answer the following questions :**

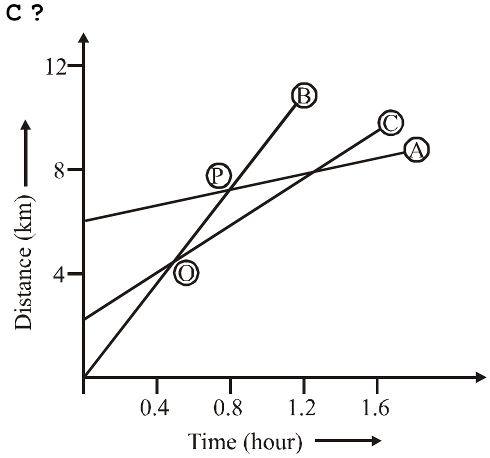
(a) Which of the three is travelling the fastest ?

(b) Are all three ever at the same point on the road ?

(c) How far has C travelled when B passes A ?

(d) How far has B travelled by the time it passes C ?

Sol. (a) Speed of car A



$$V_A = \frac{S_{final} - S_{initial}}{time} = \frac{11-6}{2} = 2.50\ km\ h^{-1}$$

Speed of car B

$$V_B = \frac{S_{final} - S_{initial}}{time} = \frac{12-0}{14} = 8.57\ km\ h^{-1}$$

Speed of car C

$$V_C = \frac{S_{final} - S_{initial}}{time} = \frac{12-2}{1.76} = 5.68\ km\ h^{-1}$$

Car B is travelling the fastest.

(b) No, they are never at the same point because all the graphs of A, B and C do not intersect at one point.

(c) When car B passes car A at point P, the distance covered by car C = 7 – 2 = 5 km.

(d) Car B and C pass each other at point O. The distance travelled by B at that point is slightly more than 5 km.

7. **A ball is gently dropped from a height of 20 m. If its velocity increases uniformly at the rate of 10 ms^{-2}, with what velocity will it strike the ground ? After what time will it strike the ground ?**

Sol. Given

Initial velocity of ball, u = 0

Final velocity of ball, v = ?

Distance through which the ball falls, S = 20 m

Acceleration a = 10 ms^{-2}

Time of fall, t = ?

We know

$v^2 - u^2 = 2aS$

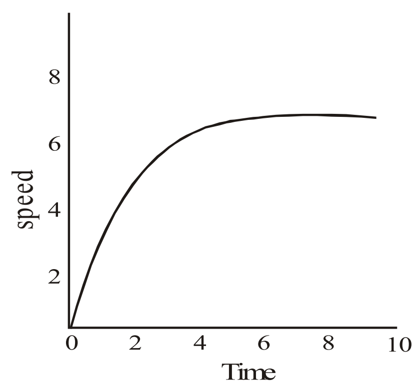
or $v^2 - 0 = 2 \times 10 \times 20 = 400$

or $v = 20\ ms^{-1}$

Now using v = u + at we have

$20 = 0 + 10 \times t$ or $t = 2s$

8. **The speed-time graph for a car is shown in figure below.**



(a) Find how far does the car travel in the first 4 seconds. Shade the area on the graph thatrepresents the distance travelled by the car during the period.

(b) Which part of the graph represents uniform motion of the car ?

Sol. a) The car travels with a non-uniform speed which is non accelerated in nature.

b) The straight line portion of the graph represents uniform motion of the car.

9. **State which of the following situations are possible and given an example for each of these**

(a) An object with a constant acceleration but with zero velocity.

(b) An object moving in a certain direction with acceleration in the perpendicular direction.

Sol. a) A body with a constant acceleration but with zero velocity is possible. For example, when a body is just released, its initial velocity u = 0, but acceleration g = 10 ms^{-2}.

b) When a stone, tied to a string, is whirled in a circular path, the acceleration acting on it is always at right angles to the direction of motion of stone.

10. **An artifical satellite is moving in a circular orbit of radius 42250 km. Calculate its speed if it takes 24 hours to revolve around the earth.**

Sol. Distance covered by the satellite in 24 hour.

$S = 2\pi r$

$= 2 \times \frac{22}{7} \times 42250 = 265571.43$ km

Therefore speed of satellite

$$v = \frac{\text{distance travelled}}{\text{time taken}} = \frac{265571.43}{24 \times 60 \times 60} = 3.07\ kms^{-1}$$

HIGHER ORDER THINKING SKILL

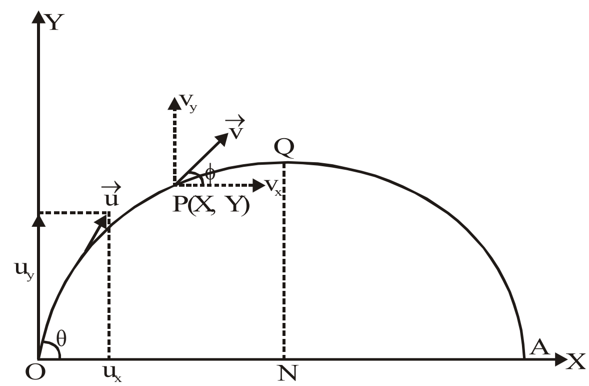
PROJECTILE'S MOTION

A projectile is an object moving in space (or air) under the effect of gravitational effect of earth alone (without any other external force) is called the projectile motion and the object is caled the projectile.

The examples of projectile are missile shot from a gun, a bomb released from an airplane, a batted cricket ball, a ball thrown at some angle with horizontal and a rocket after its fuel is exhausted.

The motion of a projectile may always be resolved into two perpendicular straight line motions, viz, horizontal and vertical motions. These motions in perpendicular directions are quite independent of each other.

Path of Projectile :-



Consider a body is projected with velocity $\vec{u}$, making an angle θ, point of projection O as the origin the axis OX and OY being horizontal and vertical directions respectively. The initial velocity $\vec{u}$ may be resolved into horizontal and vertical components.

Horizontal Component $\quad u_x = u \cos\theta$

Vertical Component $\quad u_y = u \sin\theta$

The trajectory of the projectile is parabolic.

Time of flight T :-

The time in which the porjectile again meets the horizontal plane is called the time of flight. The net vertical displacement of projectile in time of flight is zero (i.e. y = 0) ; therefore, time of flight (T) of projectile from the relation $s = ut + \frac{1}{2}at^2$ is given by :-

$$T = \frac{2u_y}{g} = \frac{2u\sin\theta}{g}$$

Maximum height H :-

At maximum height vertical component of projectile's velocity is zero, i.e., $v_y = 0$

∴ from relation $v^2 = u^2 + 2as$, we have

$$H = \frac{u_y^2}{2g} = \frac{(u\sin\theta)^2}{2g} = \frac{u^2\sin^2\theta}{2g}$$

This equation shows that the height H is maximum when $\sin\theta = 1$ or $\theta = \frac{\pi}{2}$. That is why the athlete in high jump tries to throw his body vertically upward.

Range of Projectile :-

The horizontal distance traversed by the projectile in time of flight T is called the range of projectile.

∴ Range R = horizontal speed × time of flight = $u_x T$

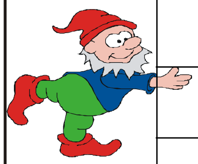
$$= u\cos\theta \cdot \frac{2u\sin\theta}{g} = \frac{u^2(2\sin\theta\cos\theta)}{g} = \frac{u^2\sin 2\theta}{g} \qquad (\because \sin 2\theta = 2\sin\theta\cos\theta)$$

For maximum range $\sin 2\theta = 1$ or $2\theta = 90°$ or $\theta = 45°$

and the maximum range, $\boxed{R_{max} = \frac{u^2}{g}}$

Obviously the maximum range is achieved when angle of projection is 45°.

DO YOU KNOW ?



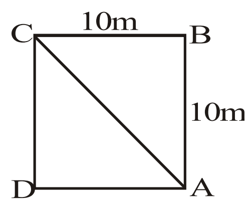
If on X-axis	& on Y-axis	then Slope	Formula	
Time	Displacement	Velocity	$\vec{v} = \frac{d\vec{s}}{dt}$	$\vec{v}_{av} = \frac{\vec{s}_f - \vec{s}_i}{time}$
Time	Velocity	Acceleration	$\vec{a} = \frac{d\vec{v}}{dt}$	$\vec{a}_{av} = \frac{\vec{v}_f - \vec{v}_i}{time}$
Time	Momentum	Force	$\vec{F} = \frac{d\vec{p}}{dt}$	$P_{av} = \frac{W}{time}$
Time	Energy	Power	$P = \frac{dW}{dt}$	$\vec{\tau}_{av} = \frac{\vec{J}_f - \vec{J}_i}{time}$

SOLVED EXEMPLES

1. **A farmer moves along the boundary of a square field of side 10 m in 40 s. What will be the magnitude of displacement of the farmer at the end of 2 minutes 20 seconds ?**

Sol. Distance covered in one round = 40 m

Time taken = 40s



Total time of journey = 2 min. 20s = 140 s

Total distance covered = $\frac{40m}{40s} \times 140s$ = 140 M

Total rounds = $\frac{140m}{40m}$ = 3.5 round

After every complete round, farmer will be at the starting point, hence after 3.5 rounds farmer will be at the diagonal ends of the field.

$$\text{Displacement} = \sqrt{(10)^2 + (10)^2} = 10\sqrt{2}\ m$$

2. **In a long distance race the athletes were expected to take four rounds of the track such that the line of finish was same as the line of start. Suppose the length of the track was 200 m**

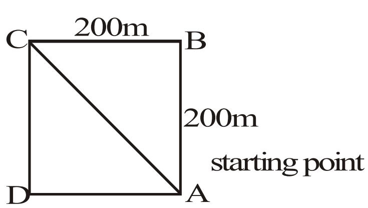
   (i) What is the total distance to be covered by the athletes?

   (ii) What is the displacement of the athletes when they touch the finish line?

   (iii) Is the motion of the athletes uniform or non-uniform.

   (iv) Is the displacement of an athlete and the distance moved by him at the end of the race equal?

Sol.



(i) Total distance covered = 4 × 200m = 800 m

(ii) As the athletes finish at the starting line

Displacement = final position-initial positive = $r_A - r_A = 0$

(iii) Motion is non-uniform as the direction of motion of the athlete is changing while running on the track.

(iv) Displacement and distance moved are not equal.

3. On a 120km track, a train travels the first 30km with a uniform speed of 30km/h. How fast must the train travel the next 90km so as to average 60km/h for the entire trip ?

Sol. Total distance d = 120km

Average speed V_{av} = 60km/h

Total time = t = ?

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time taken}}$$

$$V_{av} = \frac{d}{t}$$

$$t = \frac{d}{V_{av}}$$ Putting the values

$$t = \frac{120km}{60km/h} = 2h \text{ .... (i)}$$

Distance travelled in first part of trip

d_1 = 30km

Speed in first part of the trip v_1 = 30km/h

Time taken in first part of trip

t_1 = ?

$$t_1 = \frac{d_1}{v_1}$$ putting the values

$$t_1 = \frac{30km}{30km/h} = 1h$$

Time taken left to complete second part of the trip

$t_2 = t - t_1 = 2 - 1 = 1h$

Distance to be covered in second part of the trip

d_2 = 90 km

required speed in second part

v_2 = ?

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

$$v_2 = \frac{d_2}{t_2} = \frac{90km}{1h} = 90 \text{ km/h}$$

4. Nisha swims in a 90 m long pool. She covers 180m in one minute by swimming from one end to the other and back along the same straigh path. Find the average speed and average velocity of Nisha.

Sol. Total distance = 180m

Total displacement = 0

Time taken t = 1min. = 60s

$$\text{Average speed } (V_{av}) = \frac{\text{total distance}}{\text{total time taken}}$$

$$V_{av} = \frac{180m}{60s} = 3 \text{ m/s}$$

$$\text{Average velocity } (V_{av}) = \frac{\text{total displacement}}{\text{total time taken}}$$

$$V_{av} = \frac{0}{60s} = 0 \text{ m/s}$$

5. A bus decreases its speed from 80 km/h to 60 km/h is 5 second. Find the acceleration of the bus.

Sol. $u = 80 \text{ km/h} = 80 \times \frac{5}{18} = 22.22 \text{ m/s}$

$v = 60 \text{ km/h} = 60 \times \frac{5}{18} = 16.67 \text{ m/s}$

time t = 5s

$$\text{Acceleration (a)} = \frac{v - u}{t}$$

$$a = \frac{(16.67 - 22.22) \text{m/s}}{5\text{s}} = -1.11 \text{ m/s}^2$$

6. A train starting from a railway station and moving with uniform acceleration, attains a speed 40 km/h in 10 minutes. Find its acceleration.

Sol. u = 0 (starting from rest)

$v = 40 \text{ km/h} = 40 \times \frac{5}{18} = 11.11 \text{ m/s}$

time t = 10 minutes = 600s

$$\text{Acceleration (a)} = \frac{v - u}{t}$$

$$a = \frac{11.11 \text{m/s}}{600\text{s}} = 0.0185 \text{ m/s}^2$$

7. A bus between Kota to Jaipur passed the 100 km, 160 km and 220 km points at 10.30 am, 11.30 am and 1.30 pm. Find the average speed of the bus during each of the following intervals :

(a) 10.30 am to 11.30 am, (b) 11.30 am to 1.30 pm and (c) 10.30 am to 1.30 pm.

Sol. (a) The distance covered between 10.30 am and 11.30 am is 160 km – 100 km = 60 km. The time interval is 1 hour. The average speed during this interval is –

$$v_1 = \frac{60\text{km}}{1\text{h}} = 60 \text{ km/h}$$

(b) The distance covered between 11.30 am and 1.30 pm is 220 km – 160 = 60 km. The time interval is 2 hours. The average speed during this interval is –

$$v_2 = \frac{60\text{km}}{2\text{h}} = 30 \text{ km/h}$$

(c) The distance covered between 10.30 am and 1.30 pm is 220 km – 100 km = 120 km. The time interval is 3 hours. The average speed during this interval is –

$$v_3 = \frac{120\text{km}}{3\text{h}} = 40 \text{ km/h}$$

8. The average speed of a bicycle, an athlete and car are 18 km/h, 7 m/s and 2 km/min. respectively. Which of the three is the fastest and which is the slowest ?

Sol. $18\text{km/h} = \frac{18\text{km}}{1\text{h}} = \frac{18000\text{m}}{3600\text{s}} = 5 \text{ m/s}$

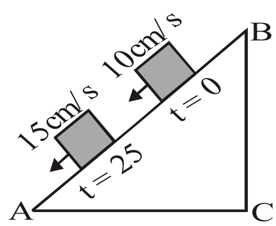
$2 \text{ km/min} = \frac{2\text{km}}{1\text{min.}} = \frac{2000\text{m}}{60\text{s}} = 33.3 \text{ m/s}$

Thus the average speeds of the bicycle, the athlete and the car are 5 m/s, 7 m/s and 33.3 m/s respectively. So the car is the fastest and the bicycle is the slowest.

9. An object is sliding down on inclined plane. The velocity changes at a constant rate from 10 cm/s to 15 cm/s in 2 seconds. What is its acceleration?

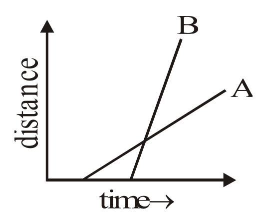
Sol. The situation is shown in figure. Let us take BA as the positive direction. The velocity at t = 0 is u = +10 cm/s and that at t = 25 is v = +15 cm/s.

Thus, $a = \frac{v-u}{t} = \frac{15 cm/s - 10 cm/s}{2 sec} = \frac{5 cm/sec}{2 sec} = 2.5 \text{ cm/s}^2$



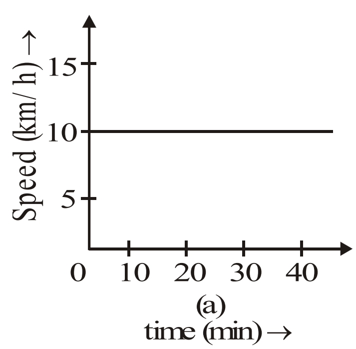
The acceleration is positive, which means it is in the direction BA.

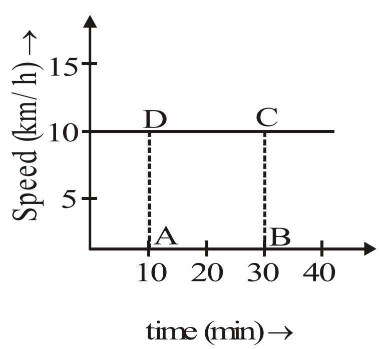
10. Figure shows distance-time graph of two objects A and B which object is moving with greater speed when both are moving?



Sol. The line for object B makes a longer angle with the time-axis. Its slope is longer than the slope of the line for object A. Thus, the speed of B is greater than that of A.

11. Figure represents the speed time graph for a particle. Find the distance covered by the particle between t = 10 min. and t = 30 min.





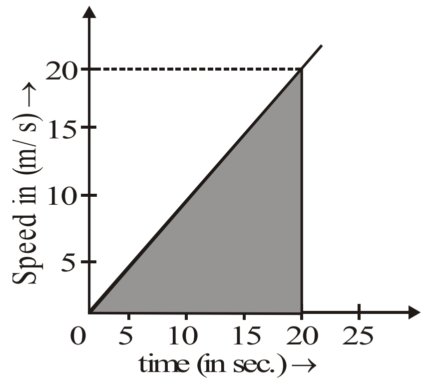
Sol. We draw perpendicular lines from the 10-minute point and the 30-minute point to the time-axis (fig.) The distance covered is equal to the area of the rectangle ABCD its value is

ABCD= (30 min.–10 min.) × (10 km/h)

= 20 min. × 10 km/h

$= \frac{20}{60}$ h × 10 km/h = $\frac{10}{3}$ km.

12. Find the distance coverd by a particle during the time interval t = 0 to t = 20 s for which the speed-time graph is shown in figure.

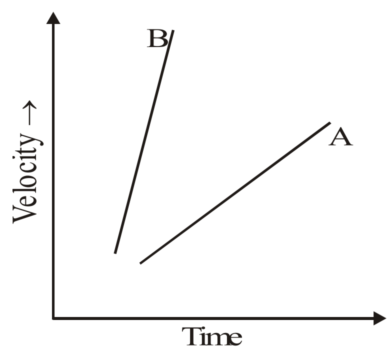


Sol. The distance covered in the time interval 0 to 20 s. is equal to the area of the shaded triangle. It is

$$\frac{1}{2} \times \text{base} \times \text{height}.$$

$$= \frac{1}{2} \times (20\ \text{s}) \times (20\ \text{m/s}) = 200\ \text{m}.$$

13. Figure shows the velocity-time graphs for two objects, A and B, moving along the same direction. Which object has greater acceleration ?

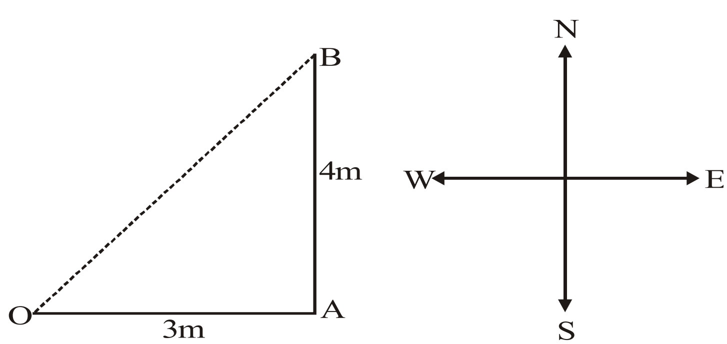


Sol. The slope of the velocity-time graph of B is greater than that for A, Thus, the acceleration of B is greater than that of A.

14. A particle moves through a distance of 3 m due east and then 4m due north.

(a) How much is the net distance traversed ? (b) What is the magnitude of the net displacement?

Sol. The situation shown in figure. The particle starts from O. It moves through a distance of 3m due east to reach A and then through a distance of 4m due north to reach B.



a) The total distance moved is 3m + 4m = 7 m,

b) The magnitude of the net displacement is OB. In the right-angled triangle OAB,

$$OB^2 = OA^2 + AB^2 = (3m)^2 + (4m)^2 = 9m^2 + 16m^2 = 25m^2 \textbf{ OR } OB = 5\ m,$$

15. A car moves 30 km. in 30 min. and the next 30 km. in 40 min. calculate the average speed for the entire journey.

Sol. Given, the total time taken is 30 min. + 40 min. = 70 min. and the total distance traversed is –

30 km + 30 km = 60 km. The average speed is

$$v_{av} = \frac{60\,\text{km}}{70\,\text{min}} = \frac{60\,\text{km}}{(70/60)\text{h}} = \frac{3600}{70}\ \text{km/hr} = 51.4\ \text{km/h}$$

16. A boy runs for 10 min. at a uniform speed of 9 km/h. At what speed should he run for the next 20 min. So that the average speed comes to 12 km/h?

Sol. Total time = 10 min + 20 min = 30 min.

The average speed is 12km/h. using s = vt, the distance covered in 30 min is

$$12\ \text{km/h} \times 30\text{min} = \frac{12\,\text{km}}{\text{h}} \times \frac{1}{2}\text{h} = 6\text{km}$$

The distance covered in the first 10 min is –

$$9\ \text{km/h} \times 10\ \text{min.} = \frac{9\,\text{km}}{\text{h}} \times \frac{1}{6}\text{h} = 1.5\ \text{km}$$

Thus, he has to cover 6 km – 1.5 km = 4.5 km. in the next 20 min. The speed required is

$$\frac{4.5\,\text{km}}{20\,\text{min}} = \frac{4.5\,\text{km}}{(20/60)\text{h}} = 13.5\ \text{km/h}$$

17. A particle was at rest from 9:00 am to 9:30 am. It moved at a uniform speed of 10 km/h from 9:30 am to 10:00 am. Find the average speed between (a) 9:00 am and 10:00 am (b) 9:15 am and 10:00 am

Sol. (a) The distance moved by the particle between 9:30 am and 10:00 am is

$$s = vt = \frac{10\,\text{km}}{\text{h}} \times \frac{1}{2}\text{h} = 5\ \text{km}.$$

This is also the distance moved between 9:00 am and 10:00 am. Thus, the average speed during this interval is

$$v_{av} = \frac{s}{t} = \frac{5\,\text{km}}{1\,\text{h}} = 5\ \text{km/h}$$

(b) The distance moved between 9:30 am and 10:00 am is 5km. This is also the distance moved in the interval 9:15 am to 10:00 am. The average speed during this interval is –

$$v_{av} = \frac{s}{t} = \frac{5\,\text{km}}{45\,\text{min}} = \frac{5 \times 60}{45}\ \text{km/h} \approx 6.67\ \text{km/h}.$$

18. A boy is running on a straight road. He runs 500 m towards north in 2.10 minutes and then turns back and runs 200 m in 1.00 minute. Calculate :-

(a) His average speed and magnitude of average velocity during first 2.10 minutes, and

(b) His average speed and magnitude of average velocity during the whole journey.

Sol. (a) Total distance = 500 m

Total time = 2.10 minutes = 130 s

Magnitude of displacement = 500 m

$$\text{Average speed} = \frac{\text{Distance}}{\text{Time}} = \frac{500\text{m}}{130\text{s}} = 3.85\ \text{ms}^{-1}$$

$$\text{Magnitude of average velocity} = \frac{\text{Magnitude of displacement}}{\text{Time}} = \frac{500\text{m}}{130\text{s}} = 3.85\ \text{ms}^{-1}$$

Note : This example shows that the average speed = average velocity if the motion is in one direction.

b) Total distance = 500 m + 200 m = 700 m

Total time = 2.10 + 1.00 = 3.10 minutes = 190 s

Magnitude of total displacement = 500 m − 200 m = 300 m

$$\therefore \quad \text{Average speed} = \frac{\text{Total displacement}}{\text{Total time}} = \frac{700\text{m}}{190\text{s}} = 3.68 \text{ ms}^{-1}$$

$$\text{Magnitude of average velocity} = \frac{\text{Total displacement}}{\text{Total time}} = \frac{300\text{m}}{190\text{s}} = 1.58 \text{ ms}^{-1}$$

Note : This example shows that average speed is greater than the magnitude of average velocity if the direction of motion changes.

19. It is estimated that the radio signal takes 1.27 seconds to reach the earth from the surface of the moon. Calculate the distance of the moon from the earth. Speed of radio signal = 3×10^8 ms^{-1} (speed of light in air).

Sol. Here, time = 1.27 s

speed = 3×10^8 ms^{-1}

distance = ?

Using distance = speed × time, we get

distance = 3×10^8 ms^{-1} × 1.27 s = 3.81×10^8 m = $3.81 = 10^5$ km

20. A wireless signal is sent to earth from a spacecraft. This signal reaches the earth in 300 seconds. Calculate the distance of the spacecraft from the earth. Given, speed of the signal = 3×10^8 ms^{-1}.

Sol. Here, t = 300 s

Speed, v = 3×10^8 ms^{-1}

Distance = ?

Using distance = speed × time, we get

distance = 3×10^8 ms^{-1} × 300 s = 9×10^{10} m = $9 = 10^7$ km

Thus, distance of spacecraft from earth = 9×10^7 km.

21. A sound is heard 5 seconds later than the lightning is seen in the sky on a rainy day. Find the distance of the location of lightning. Given speed of sound = 346 ms^{-1}.

Sol. Here, t = 5 s

Speed, v = 346 ms^{-1}

Distance = ?

Using distance = speed × time, we get

distance = 346 ms^{-1} × 5 s = 1730 m

Thus, distance of the location of lightning = 1730 m.