Quadrilaterals Ex-8.2 (solved exercise) Class-9 by-Ashish Jha

Question 1.

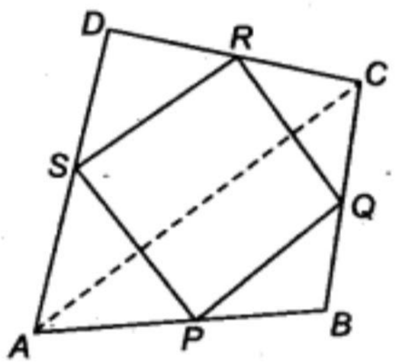
ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA (see figure). AC is a diagonal. Show that

(i) SR || AC and SR = 12 AC

(ii) PQ = SR

(iii) PQRS is a parallelogram.

Solution:



(i) In ∆ACD, We have

∴ S is the mid-point of AD and R is the mid-point of CD.

SR = 12AC and SR || AC …(1)

[By mid-point theorem]

(ii) In ∆ABC, P is the mid-point of AB and Q is the mid-point of BC.

PQ = 12AC and PQ || AC …(2)

[By mid-point theorem]

From (1) and (2), we get

PQ = 12AC = SR and PQ || AC || SR

⇒ PQ = SR and PQ || SR

(iii) In a quadrilateral PQRS,

PQ = SR and PQ || SR [Proved]

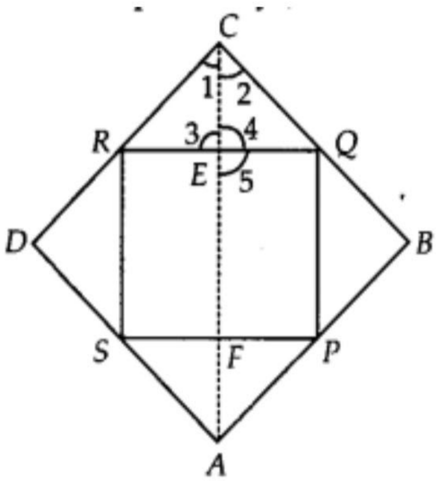
∴ PQRS is a parallelogram.

Question 2.

ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA, respectively. Show that the quadrilateral PQRS is a rectangle.

Solution:

We have a rhombus ABCD and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Join AC.



NCERT Solutions for Class 9 Maths Chapter 8 Quadrilaterals Ex 8.2 A2
In ∆ABC, P and Q are the mid-points of AB and BC respectively.
∴ PQ = 12AC and PQ || AC …(1)
[By mid-point theorem]
In ∆ADC, R and S are the mid-points of CD and DA respectively.
∴ SR = 12AC and SR || AC …(2)
[By mid-point theorem]
From (1) and (2), we get
PQ = 12AC = SR and PQ || AC || SR
⇒ PQ = SR and PQ || SR
∴ PQRS is a parallelogram. …….(3)
Now, in ∆ERC and ∆EQC,
∠1 = ∠2
[∵ The diagonals of a rhombus bisect the opposite angles]
CR = CQ [∵CD2 = BC2]
CE = CE [Common]
∴ ∆ERC ≅ ∆EQC [By SAS congruency]
⇒ ∠3 = ∠4 …(4) [By C.P.C.T.]
But ∠3 + ∠4 = 180° ……(5) [Linear pair]
From (4) and (5), we get
⇒ ∠3 = ∠4 = 90°
Now, ∠RQP = 180° – ∠b [Y Co-interior angles for PQ || AC and EQ is transversal]
But ∠5 = ∠3
[∵ Vertically opposite angles are equal]
∴ ∠5 = 90°
So, ∠RQP = 180° – ∠5 = 90°
∴ One angle of parallelogram PQRS is 90°.
Thus, PQRS is a rectangle.

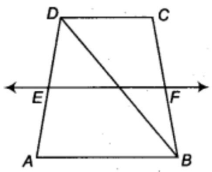
Question 3.

ABCD is a rectangle and P, Q, R ans S are mid-points of the sides AB, BC, CD and DA, respectively. Show that the quadrilateral PQRS is a rhombus.

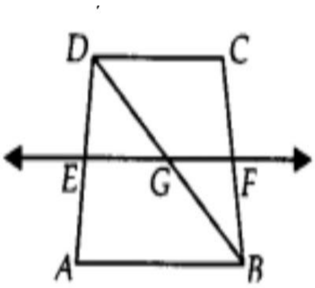
Solution:
We have,
Now, in ∆ABC, we have

PQ = 12AC and PQ || AC …(1)
[By mid-point theorem]
Similarly, in ∆ADC, we have
SR = 12AC and SR || AC …(2)
From (1) and (2), we get
PQ = SR and PQ || SR
∴ PQRS is a parallelogram.
Now, in ∆PAS and ∆PBQ, we have
∠A = ∠B [Each 90°]
AP = BP [∵ P is the mid-point of AB]
AS = BQ [∵ 12AD = 12BC]
∴ ∆PAS ≅ ∆PBQ [By SAS congruency]
⇒ PS = PQ [By C.P.C.T.]
Also, PS = QR and PQ = SR [∵opposite sides of a parallelogram are equal]
So, PQ = QR = RS = SP i.e., PQRS is a parallelogram having all of its sides equal.
Hence, PQRS is a rhombus.

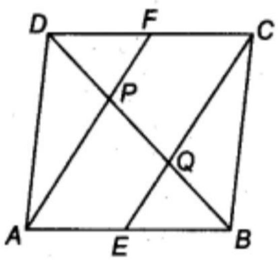
Question 4.
ABCD is a trapezium in which AB || DC, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F (see figure). Show that F is the mid-point of BC



Solution:
We have,



In ∆DAB, we know that E is the mid-point of
AD and EG || AB [∵ EF || AB]
Using the converse of mid-point theorem, we get, G is the mid-point of BD.
Again in ABDC, we have G is the midpoint of BD and GF || DC.
[∵ AB || DC and EF || AB and GF is a part of EF]
Using the converse of the mid-point theorem, we get, F is the mid-point of BC.
Question 5.
In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (see figure). Show that the line segments AF and EC trisect the diagonal BD.
Solution:



Since, the opposite sides of a parallelogram are parallel and equal.
∴ AB || DC
⇒ AE || FC …(1)
and AB = DC
⇒ 12AB = 12DC
⇒ AE = FC …(2)
From (1) and (2), we have
AE || PC and AE = PC
∴ ΔECF is a parallelogram.
Now, in ΔDQC, we have F is the mid-point of DC and FP || CQ
[∵ AF || CE]
⇒ DP = PQ …(3)
[By converse of mid-point theorem] Similarly, in A BAP, E is the mid-point of AB and EQ || AP [∵AF || CE]
⇒ BQ = PQ …(4)
[By converse of mid-point theorem]
∴ From (3) and (4), we have
DP = PQ = BQ
So, the line segments AF and EC trisect the diagonal BD.